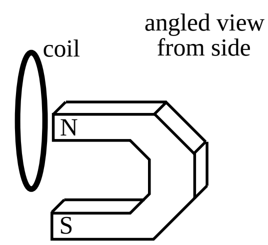
CHAPTER 17 -- MAGNETIC INDUCTION

QUESTION & PROBLEM SOLUTIONS

17.1) What is magnetic flux? How is it defined? What does it do?

Solution: Any vector field that passes through a surface will produce a flux through that surface. If a constant magnetic field passes through the face of a coil of area A, for instance, the magnetic flux through the coil will equal $\boldsymbol{B}\cdot\boldsymbol{A}$, where $\boldsymbol{A}$ is an area vector whose magnitude is equal to the area of the coil's face and whose direction is perpendicularly outward from the face. If anything varies (i.e., the magnitude or direction of $\boldsymbol{B}$ or $\boldsymbol{A}$, or the angle between the two vectors), then a differential approach must be used to determine the net flux. Mathematically, that would be $\int \boldsymbol{B}\cdot d\boldsymbol{A}$.

17.2) A coil is placed in the vicinity of a horseshoe magnet.



a.) Once in place, is there a flux through the coil?

Solution: The magnetic field lines associated with a horseshoe magnet billow out leaving the *north pole* and entering the *south pole*. That means that magnetic field lines are passing through the coil's face and there is a magnetic flux through the coil.

b.) Once in place, is there a current in the coil? If so, why? Also, if so, in what direction will the current flow?

Solution: There is nothing that would motivate charge to flow in this situation (remember, magnetic fields don't act like electric fields), so there would be no current in this situation.

17.3) The coil alluded to in *Problem 17.2* is placed in the vicinity of the same horseshoe magnet, but this time the coil is rapidly pulled away from the magnet.

a.) Is there an initial flux through the coil?

Solution: As was the case in *Problem 2a*, there is an initial magnetic flux through the coil.

b.) What happens to the flux as the coil is pulled away?

Solution: Pulling the coil away from the magnet decreases the magnetic field through the coil's face. This, in turn, decreases the magnetic flux through the coil.

c.) From the standard perspective associated with magnetic fields and charges moving in magnetic fields, would you expect a current to flow in the coil as the coil was pulled away from the magnet? If so, why? Also, in what direction would the current flow?

Solution: As far as the charges carried in the wire are concerned, they are moving across magnetic field lines as the coil is pulled away from the magnet. When this happens, they will experience a force (i.e., ***qvxB***) resulting in a current in the coil. The direction of the current can be determined using the right-hand rule. It will be clockwise as viewed from the perspective shown in the sketch.

d.) From Faraday's perspective, would you expect a current to flow in the coil as the coil was pulled away from the magnet? If so, how would Faraday explain the current? Also, how would he determine the direction of current flow?

Solution: According to Faraday, a changing magnetic flux will induce an EMF in the coil. That EMF will produce a current whose magnetic field will either *add to* or *subtract from* the external field (i.e., the field produced by the magnet). What determines which happens depends upon how the magnetic flux is changing. If it is increasing, the current will flow in such a way as to produce an induced *B-field* which will *fight the increase* by subtracting from the external field. If the magnetic flux is decreasing, the current will flow so as to create an induced *B-field* that adds to the external field, thereby *diminishing the decrease*. In all cases, the current's B-field will produce a magnetic flux of its own that will OPPOSE the *change of flux* that started the process in the first place. In this case, the magnetic flux is decreasing. A current in the clockwise direction will produce a *B-field* that adds to the external field. That will be the direction of the induced current. Note that this may seem a lot more complex than the explanation given in *Part c*, but there will be situations in which this approach/perspective is much cleaner and easier to deal with than the classical view.

17.4) Each of the loops in Figure II are identical. Each has a length of .2 meters, a width of .08 meters, and a resistance of 4 ohms. Each is moving with a velocity magnitude of .28 m/s, and *Loops A, C,* and *F* each have .05 meters of their lengths *not in the magnetic field* at the time shown in the sketch (that is, the length *outside the field* at the time shown is .05 meters for each of those loops). The magnetic field

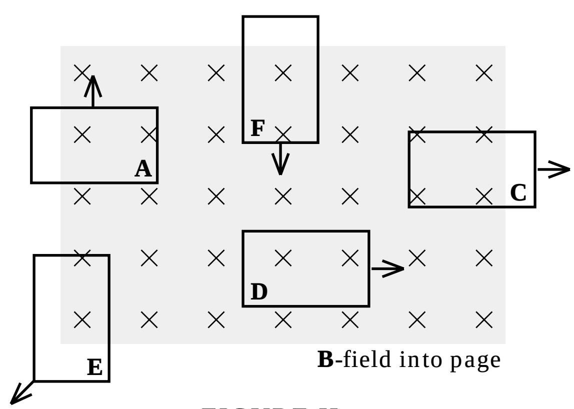


FIGURE II

in the shaded region is *into the page* with a magnitude of $B = 3x10^{-2}$ *teslas*.

a.) What is the direction of the induced current for each loop at the instant shown in the sketch?

Solution: Using Lenz's Law:

--Loop A: No induced current as there is NO CHANGING FLUX.

--Loop C: The external flux is decreasing. A CLOCKWISE induced current will produce an induced **B**-field INTO the page *through the coil's face*, which in turn will produce an induced magnetic flux that will OPPOSE the decreasing external flux.

--Loop D: No current as there is NO CHANGING FLUX.

--Loop E: The external flux is decreasing. A CLOCKWISE current will generate an induced flux that will OPPOSE the decreasing external flux.

--Loop F: The externally produced flux is increasing. A COUNTER-CLOCKWISE current will produce an induced flux that will OPPOSE the increasing external flux.

b.) What is the induced EMF generated in *Loops A, C,* and *F* at the instant shown?

Solution: Using Faraday's Law:

--Loop A: As there is no changing magnetic flux, the induced EMF in that coil will be ZERO.

--Loop C: We need to determine the loop's area change ΔA over a given amount of time Δt. In general, if the loop travels a distance *d moving with velocity v*, we can write:

$$v = d/\Delta t \quad \Rightarrow \quad \Delta t = d/v.$$

To travel, say, .05 meters going .28 m/s, it will take time:

$$\begin{aligned} \Delta t &= (.05\text{m})/(.28 \text{ m/s}) \\ &= .179 \text{ seconds.} \end{aligned}$$

During that time the area of the coil *inside the magnetic field* goes from $A_o = (.08 \text{ m})(.15 \text{ m})$ to $A_f = (.08 \text{ m})(.1 \text{ m})$, or $\Delta A = A_f - A_o = (.08 \text{ m}) (-.05 \text{ m}) = -4x10^{-3} \text{ m}^2$. We know that the induced EMF will equal:

$$\begin{aligned} \varepsilon_c &= -N_c[\Delta \phi_m/\Delta t] \\ &= -N_c [B(\Delta A)(\cos 0^o)/\Delta t] \\ &= -(1)\left[(3x10^{-2} \text{ T}) (-4x10^{-3} \text{ m}^2) (1) / (.179 \text{ s})\right] \\ &= 6.7x10^{-4} \text{ volts.} \end{aligned}$$

According to the current direction we determined in *Part a*, a *positive induced EMF* evidently corresponds to a clockwise induced current.

--Loop F: Following logic similar to that used on *Loop C*, and noting that in this case the *change of area* goes from $(.08 \text{ m})(.15 \text{ m})$ to $(.08 \text{ m}) (.2 \text{ m})$, or $\Delta A = 4x10^{-3} \text{ m}^2$, we can write:

$$\varepsilon_F = -N_F[\Delta\phi_m/\Delta t]$$
$$= -N_F\,[B(\Delta A)(\cos 0^o)/\Delta t]$$
$$= -(1)\left[(3\text{x}10^{-2}\text{ T})\,(4\text{x}10^{-3}\text{ m}^2)\,(1)\,/\,(.179\text{ s})\right]$$
$$= -6.7\text{x}10^{-4}\text{ volts.}$$

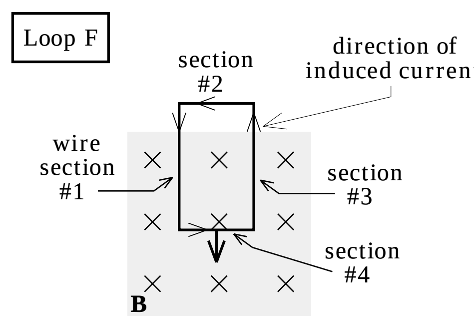
As the positive EMF in *Loop C* corresponds to a CLOCKWISE current, the negative EMF in *Loop F* should correspond to a COUNTERCLOCKWISE induced current. According to *Part a*, that is exactly what happens.

Note: The EMF in *Loop C* and in *Loop F* have the same *magnitude* because the *change of flux* during the .179 second time interval was the same in both cases.

c.) What is the magnitude and direction of the induced magnetic force felt by *Loop F* at the instant shown?

Solution: To get the force on a current-carrying wire that is bathed in a magnetic field, we must apply the expression:

$$\mathbf{F} = i\mathbf{L}\text{x}\mathbf{B}$$



to each section of the wire in the **B**-field (see figure), then add up all the forces acting as shown below:

$$\mathbf{F}_{net} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4.$$

We know that the magnitude of ***L*** in, say, *wire section #1* is equal to that portion of the wire in the **B**-field, or .15 meters. We also know that the *magnitude* of ***B*** is $3\text{x}10^{-2}$ teslas. What we don't know is the magnitude of the induced current *i*. To get that, we must determine the induced EMF, then use $i = \varepsilon_B/R$. Using the MAGNITUDE of the induced EMF from *Part b* (we just want the magnitude for the current calculation--we already know the current's direction is counterclockwise from *Part a*), we get:

$$i = \varepsilon_B/R$$
$$= (6.7\text{x}10^{-4}\text{ v})/(4\ \Omega)$$
$$= 1.68\text{x}10^{-4}\text{ amps.}$$

Noting that there is no magnetic force being applied to *wire section #2* because it is not *in* the magnetic field, we get:

$$\begin{aligned}\mathbf{F}_{net} &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4\\ &= iL_1B\sin 90^o(+\mathbf{i}) + 0 + iL_3B\sin 90^o(-\mathbf{i}) + iL_4B\sin 90^o(+\mathbf{j})\\ &= iL_4B\sin 90^o(\mathbf{j})\\ &= (1.68\text{x}10^{-4}\text{ A})(.08\text{ m})(3\text{x}10^{-2}\text{T})(1)\ (\mathbf{j})\\ &= (4.032\text{x}10^{-7}\text{ nts})\ (\mathbf{j}).\end{aligned}$$

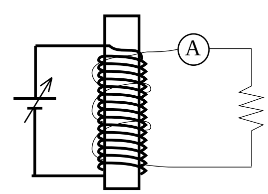
Does this make sense? Certainly! The induced force will always *oppose the motion*. As the motion is downward, the net induced force should be upward in the ***+j** direction*. That is exactly what we have calculated.
Isn't this fun?

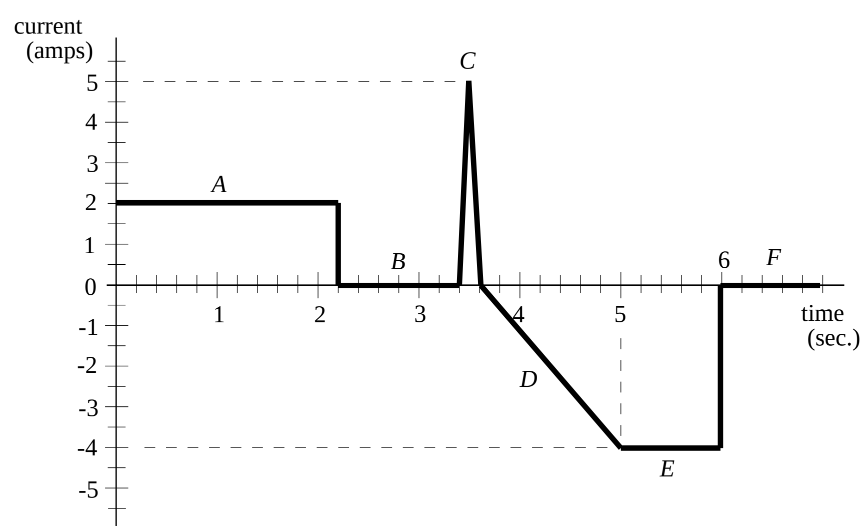
d.) What is the direction of the induced magnetic force on *Loops A, C,* and *D* at the instant shown?

Solution: The net force on *Loop A* and *Loop D* will be zero as the induced EMF in those loops is zero (hence the induced currents are zero). The induced force in *Loop C* will have the same magnitude as that of *Loop F*, but the direction will be different. How do you determine that direction?

The direction of the cross product ***iLxB*** yields the direction. Try using it. Notice that the direction of the force is always such that it *opposes the physical motion of the coil*. In the case of *Loop C*, the coil is moving in the ***+i*** direction, so the force will be in the ***-i*** direction.

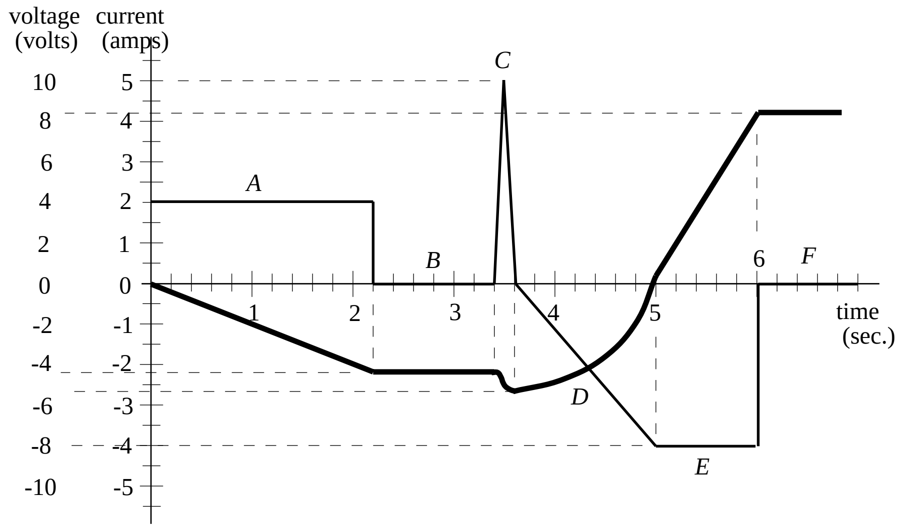
17.5) Two coils share a common axis but are electrically isolated from one another (that is, they aren't electrically connected). The coil on the left is attached to a variable power supply (we'll call this *the primary circuit*). The coil on the right is attached only to a resistor and ammeter (we'll call this *the secondary circuit*). One of the more hyperactive students in the crowd begins to play with the voltage across the primary coil power supply while a second student records, then graphs the current in the SECONDARY coil. That graph is shown in the sketch. There are six time intervals identified by letters on the graph (i.e., *A*





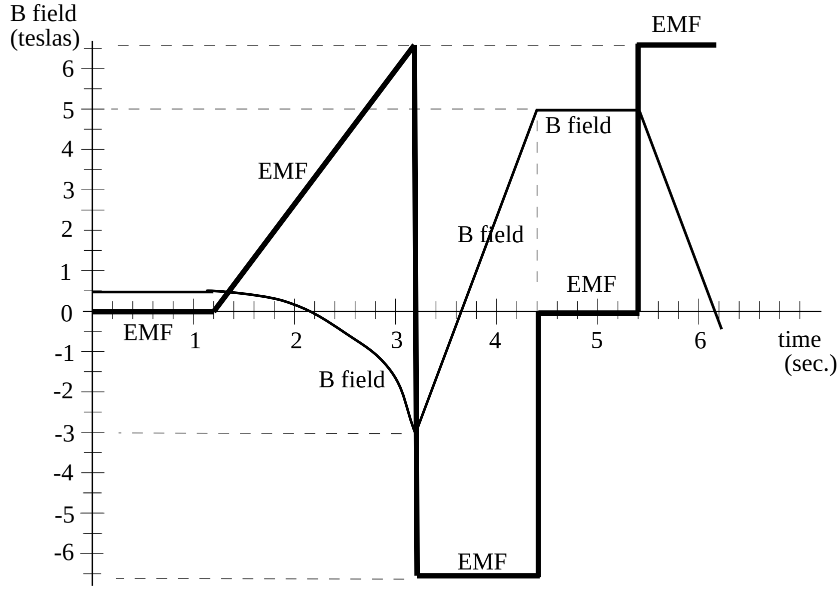
corresponds to the current during the period between $t = 0$ and $t = 2.2$ *seconds*, etc.). Explain what must be happening to the power supply in the *primary circuit* during each of those time periods.

Solution: Every time the power supply voltage changes, there is an increase or decrease of current in the primary circuit (that is, di_{prim}/dt is non-zero). That change of current through the primary coil produces a changing magnetic flux through both coils via the steel rod. With the changing flux comes an induced EMF across the secondary coil ε_{sec} which motivates current to flow in the secondary circuit (note that $\varepsilon_{sec} = i_{sec}R$ and that our graph is that of i_{sec}). We know from Faraday's Law that $\varepsilon_{sec} = -L(di_{prim}/dt)$, which further implies that $di_{prim} = -\int \varepsilon_{sec}dt$. Evidently, what is happening in the primary coil is related to the area under the *EMF*$_{sec}$ *versus time* graph, where the *EMF*$_{sec}$ *versus time* graph is proportional to the graph we have been given (i.e., i_{sec} *vs. time*). Using what we know, then, yields the final graph as shown. Note that conceptually, this all follows nicely. Think about it. If the change in the primary voltage occurs for only a moment, we will see a spike of induced current in the secondary, then nothing (where do you find a spike in our graph--what kind of primary voltage change should go with that spike?). If the primary voltage changes linearly, we will see a constant induced current in the secondary (where do you find a constant current--what kind of a primary voltage goes with *that*?). If the primary voltage changes as a quadratic, we will see an induced secondary current that increases or decreases linearly with time (where do you find that on our graph?). And how big are the voltage changes? They are proportional to the areas under the current graphs over the time periods of interest. As long as you don't get messed up with the negative sign that is inherent in Faraday's Law, it's easy!



17.6) The magnetic field down the axis of a coil varies with time as graphed to the right. On the graph, sketch the induced EMF set up in the coil.

Solution: A change in the magnetic field will change the magnetic flux down the coil's axis which will, in turn, induce an EMF across the coil. If the magnetic field is constant, there is no change in the magnetic flux and the induced EMF will be zero. If the magnetic field changes linearly, the induced EMF will be a constant. In fact,



the size of the EMF is determined using $\varepsilon = -N(d\phi_m/dt) = -NA\cos\theta\,(dB/dt)$, where N is the number of winds in the coil, A is the constant cross sectional area of the coil, and θ is the angle between $\boldsymbol{B}$ and $\boldsymbol{A}$ (it is assumed to be zero degrees in this case). In other words, the induced EMF is related to the slope of the magnetic field function (actually, it's *minus* the slope . . .). In any case, the EMF's graph is superimposed on the original *B-field* graph.

17.7) If the graph in *Problem 17.6* had been of the EMF set up in the coil as a function of time, what could you say about the magnetic flux through the coil?

Solution: In general, if the EMF is related to the slope of the magnetic field function, the magnetic field function will be related to the area under the EMF graph.

17.8) A 6-turn circular coil whose radius is .03 meters and whose net resistance is 12 Ws is placed squarely (that is, $\boldsymbol{A}$ and $\boldsymbol{B}$ are parallel to one another) in a magnetic field whose direction is *out of the page* and whose magnitude is 2.3 teslas.

a.) What is the coil's initial magnetic flux?

Solution: The magnetic flux is:

$$\phi_m = \mathbf{B}\cdot\mathbf{A}$$
$$\phi_m = BA\cos\theta$$
$$= (2.3\text{ T})[\pi(.03\text{ m})^2]\cos 0^o$$
$$= 6.5\text{x}10^{-3}\text{ webers.}$$

b.) If the field increases at a rate of .6 teslas per second, what is the magnitude and direction of the induced current in the coil?

Solution: This problem is most easily done using Calculus, so that's the way we will go. The *area* vector is not changing. The magnetic field vector is changing and we know *the rate at which that change occurs* (i.e., dB/dt). Noting that $d\phi_m = A(dB)$ and, for this case, $d\phi_m/dt = A(dB/dt)$, Faraday's Law can be written as:

$$\begin{aligned}\varepsilon &= -N\,[d\phi_m/dt] \\ &= -N\;[\quad A \qquad (dB/dt) \quad (\cos 0^o)] \\ &= -(6)\;[\pi(.03\text{ m})^2 \quad (.6\text{ T/sec}) \quad (1)\;] \\ &= -1\text{x}10^{-2}\text{ volts.}\end{aligned}$$

Using $i = \varepsilon/R$, we get a current magnitude of:

$$\begin{aligned}i &= (10^{-2}\text{ V})/(12\ \Omega) \\ &= 8.33\text{x}10^{-4}\text{ amps.}\end{aligned}$$

According to Lenz's Law, the current should flow CLOCKWISE.

c.) Go back to the original situation. The coil is made to rotate about its vertical axis at an angular frequency of ω = *55 radians per second*. That means the induced EMF is AC.

i.) What is the frequency of the AC current generated?

Solution: The frequency of the AC current will be the same as the frequency of the coil's rotation. As $\omega = 2\pi\nu$, we can write:

$$\begin{aligned}\nu &= \omega / 2\pi \\ &= (55\text{ rad/sec})/(2\pi) \\ &= 8.75\text{ hz.}\end{aligned}$$

ii.) Determine an expression for the *induced EMF* in the circuit.

Solution: Again, using Calculus: In this case, **B** and **A** are constant while the angle between the two vectors changes with time. We can write the angle as a function of time by noting that $\phi = \omega t$. With this, Faraday's Law yields:

$$\begin{aligned}\varepsilon &= -N\frac{d\phi_m}{dt} \\ &= -N\frac{d(BA\cos\omega t)}{dt} \\ &= -NBA\frac{d(\cos\omega t)}{dt} \\ &= -NBA\omega[-\sin\omega t] \\ &= NBA\omega(\sin\omega t).\end{aligned}$$

Putting in the numbers yields:

$$\varepsilon = 2.15\sin(55t)\text{ volts.}$$

17.9) For the *RL circuit* shown in Figure III, the inductance is 1.5 henrys and the inductor's internal resistance is 6 ohms. A current of 2.5 amps has been flowing in the circuit for a long time. At $t = 0$, the power is switched off and the current begins to die.

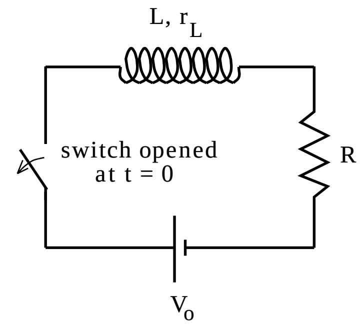


FIGURE III

a.) What is the voltage across the inductor BEFORE $t = 0$?

Solution: Before the current begins to change, the only voltage drop across the inductor is due to the *internal resistance* inherent within the inductor's wire. That means:

$$
\begin{aligned}
V_L &= ir_L \\
&= (2.5 \text{ A})(6\ \Omega) \\
&= 15 \text{ volts.}
\end{aligned}
$$

b.) After .05 seconds, the current has dropped to approximately one-third of its original value. Determine the resistance of the resistor R. (Hint: think about the *time constant* of an RL circuit and what it tells you).

Solution: The *time constant* for an inductor/resistor circuit tells us how long it takes for the current to reach .63 of its maximum (assuming the switch closes at $t = 0$) or, if the system has been *turned off* as is the case here, the time it takes for the current to FALL to .37 of its maximum. In other words, knowing that it took .05 seconds to hit approximately one-third of its original value after the switch is opened means the *time constant* for the RL circuit is approximately .05 seconds. Noting that the total resistance in the circuit is $(r_L + R)$ and remembering that the *time constant* for an RL circuit is $\tau_{RL} = L/(R + r_L)$, we can write:

$$
\begin{aligned}
& L / (R + r_L) = .05 \\
\Rightarrow \quad & R + (6\ \Omega) = [(1.5 \text{ H}) / (.05)] \\
\Rightarrow \quad & R = 24\ \Omega.
\end{aligned}
$$

c.) How much POWER does the inductor provide to the circuit over the .05 second time period alluded to in *Part b*? (Hint: Think about the definition of power and what you know about stored energy in a current-carrying inductor).

Solution: The energy stored in a current-carrying inductor is equal to $(1/2)Li^2$. If that value decreases, which it will as the current decreases, the "lost" energy goes into driving current in the circuit even longer than expected (remember, inductors are coils and coils hate to have the flux through their cross-section change). The amount of energy provided to the circuit is equal to $(1/2)Li_f^2 - (1/2)Li_o^2$ (this number will actually be negative--the negative telling you that the inductor is *losing* that amount of energy to the circuit). As POWER is defined as the *work done* (read this "energy given up") *per unit time*, then:

$$P = [(1/2)Li_f^2 - (1/2)Li_o^2]/\Delta t$$
$$= [.5(1.5\text{ H})[.33(2.5\text{ a})]^2 - .5(1.5\text{ H})(2.5\text{ a})^2]/(.05\text{ sec})$$
$$= -83.5\text{ watts.}$$

d.) The power given up by the inductor: where did it go?

Solution: The energy goes into driving current through the circuit even after the battery has been taken out of the circuit by the switch. That is, if the resistor is a light bulb, it will "burn" very bright with the initial change, then dampen out over some period of time (how long this takes depends upon the resistance in the circuit).

17.10) A rectangular coil of area A_o has N turns in it. It is rotated in a time-varying magnetic field (see Figure V) equal to $B_o e^{-kt}$, where k is a constant and B_o is the amplitude of the magnetic field. Assuming the frequency of the rotation is n:

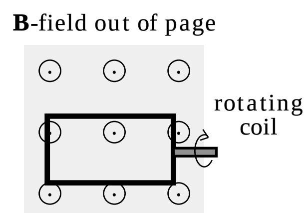


FIGURE V

a.) Determine the EMF in the coil as a function of time.

Solution: Once again, using Calculus: We can take care of the rotation by writing the time-varying angle between **A** and **B** as $\theta = \omega t$ (we can put in the $\omega = 2\pi\nu$ later). That makes the magnetic flux look like:

$$\begin{aligned}\phi_m &= BA_o\cos\theta \\ &= \left(B_o e^{-kt}\right)A_o\cos\omega t.\end{aligned}$$

The induced EMF is:

$$\begin{aligned}\varepsilon &= -N\frac{d\phi_m}{dt} \\ &= -N\frac{d\left(\left(B_o e^{-kt}\right)A_o\cos\omega t\right)}{dt} \\ &= -NB_oA_o\frac{d\left((e^{-kt})\cos\omega t\right)}{dt} \\ &= -NB_oA_o\left[(-k)(e^{-kt})\cos\omega t + (e^{-kt})\omega(-\sin\omega t)\right] \\ &= NB_oA_oe^{-kt}\left[k\cos\omega t + \omega\sin\omega t\right].\end{aligned}$$

b.) At what point in time will the magnitude of the EMF be at a maximum?
Solution: More Calculus: To determine the maximum value of the EMF, we must determine the time when the rate of change of the EMF is zero (this is a standard maximization problem). Doing this process yields:

$$\begin{aligned}\frac{d\varepsilon}{dt} &= NB_oA_o\,\frac{d\left(e^{-kt}\left[k\cos\omega t+\omega\sin\omega t\right]\right)}{dt}\\ &= NB_oA_o\,\frac{d\left(e^{-kt}\left[k\cos\omega t+\omega\sin\omega t\right]\right)}{dt}\\ &= NB_oA_o\left[(-k)e^{-kt}\left[k\cos\omega t+\omega\sin\omega t\right]+e^{-kt}\left[-k\omega\sin\omega t+\omega^2\cos\omega t\right]\right].\end{aligned}$$

The time-derivative of the EMF expression yields the *slope of the EMF function*. As maxima or minima have tangent-slopes equal to zero, we can write:

$$\begin{aligned}\frac{d\varepsilon}{dt} &= NB_oA_o\left[(-k)e^{-kt}\left[k\cos\omega t+\omega\sin\omega t\right]+e^{-kt}\left[-k\omega\sin\omega t+\omega^2\cos\omega t\right]\right]\\ &= 0.\end{aligned}$$

Canceling out the $NB_oA_oe^{-kt}$ terms and multiplying by -1, we can write:

$$\begin{aligned}&k^2\cos\omega t+\omega k\sin\omega t+k\omega\sin\omega t-\omega^2\cos\omega t=0\\ \Rightarrow\quad &(+\omega k+k\omega)\sin\omega t+(k^2-\omega^2)\cos\omega t=0\\ \Rightarrow\quad &(2\omega k)\sin\omega t=-(k^2-\omega^2)\cos\omega t\\ \Rightarrow\quad &\frac{\sin\omega t}{\cos\omega t}=\frac{(-k^2+\omega^2)}{2\omega k}\\ \Rightarrow\quad &\tan\omega t=\frac{(-k^2+\omega^2)}{2\omega k}\\ \Rightarrow\quad &\omega t=\tan^{-1}\left[\frac{(-k^2+\omega^2)}{2\omega k}\right]\\ \Rightarrow\quad &t=\frac{1}{\omega}\tan^{-1}\left[\frac{(-k^2+\omega^2)}{2\omega k}\right].\end{aligned}$$

Be impressed. The units of the *inverse tangent* are *radians*; the units of the coefficient are *seconds* (remember, because *radians* is a generic term, the units of *w* is technically *seconds*$^{-1}$). Everything seems to be working, at least as far as the units go.

17.11) A fixed circular coil of radius *R* is placed in a magnetic field that varies as $12t^3 - 4.5t^2$. If the coil has *N* winds and ***A*** is defined as *out of the page* (i.e., in the *+**k**-direction*):

field direction is time-dependent--the direction of ***A*** is out of the page

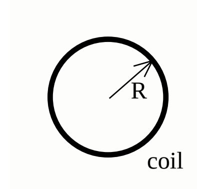


FIGURE VI

a.) Is ***B*** into or out of the page at *t = .2 seconds*?

Solution: Evaluating $B = 12t^3 - 4.5t^2$ at *t = .2 seconds* yields *B = -.084 teslas*. The negative sign implies that ***B*** is into the page at *t = .2 seconds*.

b.) Derive a *general expression* for the magnetic flux through the coil.

Solution: The general expression for the magnetic flux is:

$$\begin{aligned}\phi_m &= \mathbf{B} \bullet \mathbf{A} \\ &= BA\cos\theta \\ &= \left(12t^3 - 4.5t^2\right)(\pi R^2)\cos 0^\circ \\ &= \left(12t^3 - 4.5t^2\right)(\pi R^2).\end{aligned}$$

c.) What is the *general expression* for the induced EMF in the coil?

Solution: Again, with the Calculus: The induced EMF is:

$$\begin{aligned}\varepsilon &= -N\frac{d\phi_m}{dt} \\ &= -N\frac{d\left(\left(12t^3 - 4.5t^2\right)(\pi R^2)\right)}{dt} \\ &= -N\pi R^2\frac{d\left(12t^3 - 4.5t^2\right)}{dt} \\ &= -N\pi R^2\left[36t^2 - 9t\right].\end{aligned}$$

d.) Determine the *two* points in time when the induced EMF is zero.

Solution: The EMF will equal zero when $36t^2 - 9t = 0$. This will occur at *t = 0* and at *t = 9/36 = 1/4 = .25 seconds*.

e.) What is the direction of the current flow:

i.) Just before $t = .25$ *seconds*?

Solution: The magnetic flux *change* is what governs the direction of the induced current. Although it isn't always true, in this problem the changing flux is due solely to the changing magnetic field (***A*** and the angle between ***A*** and ***B*** are both fixed). In other words, for the *just before t = .25 seconds* part, we need to know:

--How the external magnetic field is changing just before $t = .25$ *seconds* (this will tell us if the induced magnetic field *adds to* or *subtracts from* the external magnetic field) and;

--The direction of the external magnetic field just before $t = .25$ *seconds* (this tells us in which direction the addition or subtraction must occur).

To make things easier, let's begin by graphing the magnetic field function. To do so:

--We will use the magnetic field expression given in the problem for points in time around $t = .25$ *seconds*, and;

--We can use the fact that the EMF is zero at $t = .25$ *seconds* (that means the slope of the magnetic flux must be zero at that point in time which, in this case, means the slope of the *magnetic field* expression must be zero at that point in time).

--Putting it all together, we get the graph shown in the sketch. So for just before $t = .25$ *seconds*:

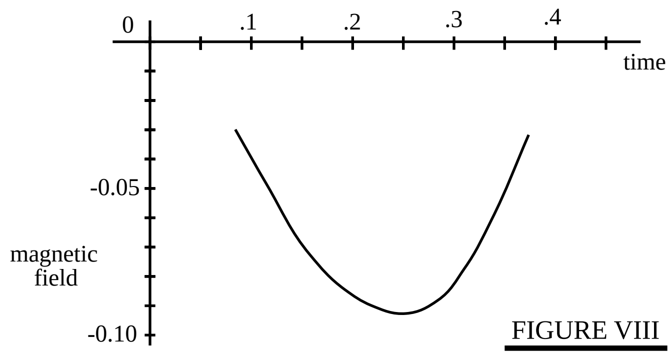


FIGURE VIII

--From the graph, the magnetic field is negative and, hence, *into the page* just before $t = .25$ *seconds*.

--The magnetic field is getting *bigger* (i.e., more negative) just before $t = .25$ *seconds*.

--An increasing magnetic field (hence, magnetic flux) will induce a current that fights the increase.

--The induced magnetic field that fights an increasing external magnetic field directed *into the page* will itself be directed *out of the page*.

--The induced current that produces such an induced field will be in the counterclockwise direction. That is the direction of the induced current before $t = .25$ *seconds*.

ii.) Just after $t = .25$ *seconds*?

Solution: --From the graph, the magnetic field is still negative and, hence, *into the page* just after $t = .25$ *seconds*.

--The magnetic field is getting *smaller* (i.e., it's proceeding back toward zero) just after $t = .25$ *seconds*.

--A decreasing magnetic field (hence, magnetic flux) will induce a current that fights the decrease.

--The induced magnetic field that fights a decreasing external magnetic field directed *into the page* will itself be directed *into the page*.
--The induced current that produces such an induced field will be in the clockwise direction. That is the direction of the induced current after *t = .25 seconds*.

Note: The EMF at *t = .2 seconds* is $-.36N\pi R^2$ while the EMF at *t = .3 seconds* is $.54N\pi R^2$. As the EMF and the *change in flux* are essentially the same, this tells us that the changes are different on either side of *t = .25 seconds* and, hence, that the induced currents will be in different directions. This is really the only generalization we can make from the EMF information.

f.) Derive the general expression for the induced electric field setup in the coil.

Solution: The last Calculus you will see: The relationship that is important here is:

$$N\frac{d\phi_m}{dt} = -\oint \mathbf{E} \bullet d\mathbf{l},$$

where $\boldsymbol{E}$ is the electric field evaluated along a differential path-length $d\boldsymbol{l}$. In most problems, the magnitude of $\boldsymbol{E}$ is assumed to be constant and in the direction of $d\boldsymbol{l}$, so the above equation becomes:

$$N\frac{d\phi_m}{dt} = -E\oint dl.$$

In this case, the integral is simply adding differential sections around a closed circular loop (i.e., the integral equals $2\pi r$, where r is the radius of the circular path). Using this, we get:

$$\begin{aligned} N\frac{d\phi_m}{dt} &= -E\oint dl \\ \Rightarrow \quad N\pi R^2\left[36t^2 - 9t\right] &= -E(2\pi R) \\ \Rightarrow \quad E &= \frac{-NR\left[36t^2 - 9t\right]}{(2)}. \end{aligned}$$

g.) An electron is placed at *R/2* in the field. Derive an expression for its acceleration at time *t = 3.3 seconds*. For this, assume *N = 15* and *R = .2 meters*.

Solution: N.S.L. maintains that $F = ma$. As the force in this case is generated by the electric field $\boldsymbol{E}$, we can also write $F = qE$. Combining the two, we get $a = qE/m$. Substituting $R/2$ for R in the general electric field expression derived above, we get:

$$
\begin{aligned}
a &= \frac{q}{m}E \\
&= \frac{q}{m}\left[\frac{-N(R/2)\left[36t^2 - 9t\right]}{(2)}\right] \\
&= \frac{(1.6\text{x}10^{-19}\text{coul})}{(9.1\text{x}10^{-31}\text{kg})}\left[\frac{-(15)(.2\text{meters}/2)\left[36(3.3\sec)^2 - 9(3.3\sec)\right]}{2}\right] \\
&= -4.78\text{x}10^{13}\ \text{m}/\text{s}^2.
\end{aligned}
$$

Note: The electric field at $t = 3.33$ *seconds* is only $4.14\text{x}10^{-3}$ nt/C. What makes this acceleration so large is the *charge to mass ratio q/m*.

17.12) The transformer shown in Figure VIII has 1200 winds in its primary coil and 25 winds in its secondary. The resistance in its primary is 80 Ωs, the resistance in its secondary is 3 Ωs, and the primary's inductance is $L_p = 10\ mH$. A 110 volt DC power supply is hooked into the primary providing an 8.25 amp current to the system. The switch has been closed for a long time. When the switch is opened, the current drops to zero in .04 seconds.

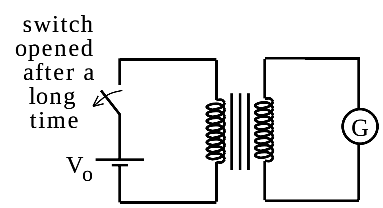


FIGURE VIII

a.) What is the induced EMF across the primary before the switch is opened?

Solution: The inductor-induced EMF across the primary when there is no current change in the circuit is zero.

b.) What is the induced EMF across the primary during the current change?

Solution: When there is a current change in the primary circuit, the inductor-induced EMF setup across the primary coil is:

$$
\begin{aligned}
\varepsilon_p &= -L\,(\Delta i/\Delta t) \\
&= -(10^{-2}\ \text{H})[(0 - 8.25\ \text{A})/(.04\ \text{sec})] \\
&= 2.06\ \text{volts}.
\end{aligned}
$$

c.) What is the current in the primary during the current change (i.e., after the switch is opened)?

Solution: The only voltage in the circuit after the switch is opened is that due to the *induced EMF* across the primary coils of the transformer. As such:

$$\begin{aligned} i_p &= \varepsilon_p/R \\ &= (2.06\text{ v})/(80\ \Omega) \\ &= .0257\text{ amps.} \end{aligned}$$

d.) What is the current in the secondary before the switch is opened?

Solution: In the secondary circuit, there is no power supply. There is also no *changing flux* before the switch is opened. Therefore, the induced EMF across the secondary will be ZERO before the switch is opened, and the induced current will also be zero.

e.) What is the current in the secondary after the switch is opened and DURING the current change?

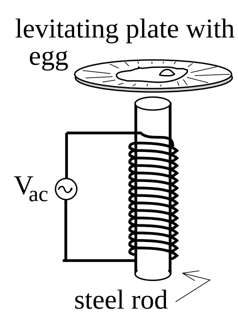
Solution: After the switch is opened, the secondary current will be such that:

$$N_p/N_s = i_s/i_p$$
$$\begin{aligned} \Rightarrow \quad i_s &= i_p N_p/N_s \\ &= (.0257\text{ amps})[1200/25] \\ &= 1.23\text{ amps.} \end{aligned}$$

f.) Is this a step-up or step-down transformer?

Solution: As $N_s < N_p$, the transformer must be a step-down type (the secondary voltage is *stepped down* relative to the primary voltage).

17.13) An AC source is attached to a coil that has a vertical, steel bar down its axis. When the power is turned on, an alternating magnetic field is set up along the axis of the bar. An aluminum plate is centered over the bar at its upper end. When power is provided to the coil, the plate levitates.



a.) Is aluminum a magnetizable material?

Solution: No! Aluminum is not like iron. Its atoms don't have more electrons spinning in one direction than the other, so aluminum atoms are not magnets unto themselves as is the case with iron atoms.

b.) Why does the plate levitate?

Solution: Although aluminum isn't magnetizable, it is a metal. As such, it has metallic bonding and it does have valence electrons that are free to move around within the structure. So what's going on within the plate? The changing magnetic field through the coil produces a changing magnetic flux through the plate. That changing flux induces an EMF that motivates free charge (electrons) in the plate to move about. The motion of those electrons sets up a magnetic field of its own (remember, charge in motion generates a magnetic field). This induced magnetic field will alternate just as does the external magnetic field produced by the coil. The difference is that the two fields will be out of phase with one another. That is, when the field produced by the coil has the upper end of the steel acting like a *north pole*, the bottom surface of the plate will be acting like a *north pole*. The repulsion between the opposing magnetic fields provides the force that levitates the plate.

c.) An egg is broken onto the plate. What will happen to the egg . . . and why?

Solution: The motion of the electrons in the plate will cause the plate to heat. If you put an egg onto the plate, it will cook. As bizarre as this sounds, the demo actually works--you can cook an egg on the levitating aluminum plate.

17.14) What is inductance? How is it comparable to resistance and capacitance?

Solution: The original presentation of Faraday's Law maintained that whenever there was a changing magnetic flux through a coil, that changing flux would be accompanied by an induced EMF. Mathematically, this was represented as $\varepsilon = -N(d\phi_m/dt)$. When dealing with coils in electrical circuits, though, it was observed that the changing magnetic flux was really being generated by a change in the current through the coil. To reflect that fact in a mathematical sense, someone decided to write Faraday's Law in terms of di/dt instead of $d\phi_m/dt$. The proportionality constant required to make the relationship work was called *the inductance* L of the coil. With that constant, Faraday's Law became $\varepsilon = -L(di/dt)$. In short, inductance is the proportionality constant that relates the induced EMF (i.e., the voltage across the coil's leads) and the current change di/dt that created the induced voltage in the first place.

It is interesting to note that all of the circuit elements you have run into so far have had defining parameters that have been proportionality constants that related the voltage across the element to either the current through the element or the charge accumulated on the element. For resistors, *resistance* was defined such that $V = iR$; for capacitors, *capacitance* was defined as $q = CV$. *Inductance* follows the pattern nicely.

17.15) How do transformers work?

Solution: A transformer is made up of two coils that are not electrically connected but that are magnetically coupled. Normally used in an AC setting, a voltage change across the coil in the primary creates a changing magnetic flux through the secondary coil (remember, the two coils are magnetically linked) which produces an EMF in the secondary coil. It is this EMF that drives current in the secondary coil.

Transformers are used primarily for transferring power from one part of an electrical circuit to another part without electrically connecting the two parts. In addition, if the number of winds N_s in the secondary coil is greater than the number of winds N_p in the primary coil, the secondary voltage will *step up* relative to the primary voltage (this is called a *step-up* transformer), with a proportional stepping down of the current. (The opposite of this is the *step-down* transformer in which $N_s < N_p$.) Europe's electrical wall sockets, for instance, run at 220 volts. For you to use your electric shaver--a device that runs on 110 volt AC--you have to use a step-down transformer to get the shaver to work properly.

17.16) You have just built from scratch a stereo system. You have 8 ohm speakers you would like to plug into the system, but as it stands the system's unrestricted output impedance is 60 ohms. Assuming you don't want to completely redesign the entire system, what would you have to do so that your set-up could run the 8 ohm speakers without power loss. Be specific and include numbers where applicable.

Solution: To match the impedance between the two units, you have to use a transformer. The relationship that governs the turns-ratio is $(N_p / N_s)^2 = Z_{st} / Z_{load}$, where Z_{load} is the true load resistance (i.e., that of the speakers) and Z_{st} is the impedance of the signal's source. Putting in the numbers for our situation, we need a transformer whose turns-ratio is:

$$\begin{aligned}(N_p / N_s) &= [Z_{st} / Z_{load}]^{1/2} \\ &= [(60\text{ ohms})/(8\text{ ohms})]^{1/2} \\ &= 2.7386\end{aligned}$$

Using a transformer whose N_p = *274 winds* and whose N_s = *100 winds* would do the job.